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Mathematics Short E-Course
Session Three: Integration
Exercise Sheet - Solutions

Solution 1

Find the general form of the following indefinite integrals:

(a)

$$\int (x^3+9)\,dx = \int (x^3+9x^0)\,dx$$

$$= \left(\frac{1}{3+1}\right)x^{3+1} + 9\left(\frac{1}{0+1}\right)x^{0+1} + c = \frac{1}{4}x^4 + 9x + c$$

(b)

$$\int (8x^7+3x^{-5}-2x)\,dx = \int (8x^7+3x^{-5}-2x^1)\,dx$$

$$= 8\left(\frac{1}{7+1}\right)x^{7+1} + 3\left(\frac{1}{-5+1}\right)x^{-5+1} - 2\left(\frac{1}{1+1}\right)x^{1+1} + c$$

$$= 8\left(\frac{1}{8}\right)x^8 + 3\left(\frac{1}{-4}\right)x^{-4} - 2\left(\frac{1}{2}\right)x^2 + c$$

$$= x^8 - \frac{3}{4}x^{-4} - x^2 + c$$

(c) $\int \sqrt[5]{t}\,dt = \int t^{1/5}\,dt = \left(\frac{1}{1/5+1}\right)t^{1/5+1} + c = \left(\frac{1}{6/5}\right)t^{6/5} + c = \frac{5}{6}t^{6/5} + c$

Solution 2

Determine $I = \int (3x^4)\,dx$ given that $I = 1\ when\ x = 1$.

$$I = \int (3x^4)\,dx = 3\left(\frac{1}{4+1}\right)x^{4+1} + c = \frac{3}{5}x^5 + c$$

$$1 = \frac{3}{5}(1)^5 + c \Rightarrow c = 1 - \frac{3}{5} = \frac{2}{5}$$

$$I = \frac{3}{5}x^5 + \frac{2}{5}$$